MIND MAP : LEARNING MADE SIMPLE

Chapter-1

Pair of Linear Equations in Two Variables

General Form

$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$

$a_1, b_1, c_1, a_2, b_2, c_2$ – Real numbers

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

Algebraic Methods

By Substitution

Solve: $7x-15y = 2$ –(i)
$x+2y = 3$ –(ii)

Solution: From equation (ii), $x = 3-2y$ -(iii)
substitute value of x in eq. (i)
$7(3-2y)-15y = 2$
$-29y = -19 \Leftrightarrow y = \frac{19}{29}$
$\therefore$ In eq. (iii) $x = 3 - 2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

By Elimination

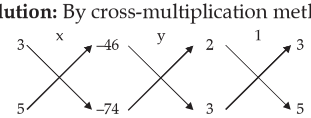
Solve: $2x+3y = 8$ – (i)
$4x+6y = 7$ – (ii)

Solution: From eq. (i)x 2–eq.(ii)x1, we have
$(4x-4x) + (6y-6y) = 16-7$
$0 = 9$, which is a false statement
The pair of equation has no solution

By Cross-Multiplication

Solve: $2x+3y-46 = 0$ –(i)
$3x+5y-74 = 0$ –(ii)

Solution: By cross-multiplication method



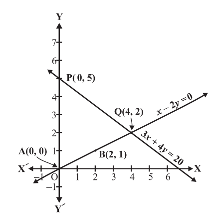
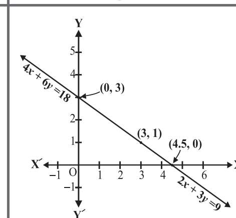
Then, $\frac{x}{3(-74)-5(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{2(5)-3(3)}$

$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$

$\frac{x}{8} = \frac{y}{10} = \frac{1}{1} \Leftrightarrow \frac{x}{8} = \frac{1}{1}$ and $\frac{y}{10} = \frac{1}{1}$

i.e. $x = 8$ and $y = 10$

Graphical Representation

S. No.	Pair of Lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the Ratios	Graphical Representation	Algebraic Interpretation
1.	$x-2y = 0$ $3x+4y-20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	 Intersecting Lines	Exactly one solution – consistent (unique)
2.	$2x+3y-9 = 0$ $4x+6y-18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$		Infinitely many solutions – Dependent

MIND MAP : LEARNING MADE SIMPLE   Chapter-2

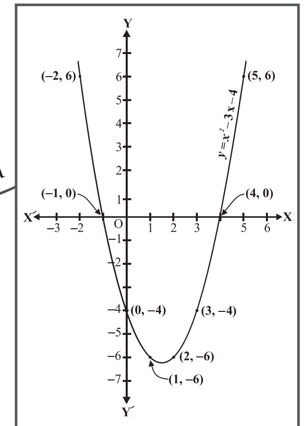
Polynomials

Division Algorithm

If $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then –
$p(x) = g(x) \times q(x) + r(x)$
where, $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$

Graphical Representation-Quadratic Polynomial

Parabola



$y = x^2 – 3x – 4$

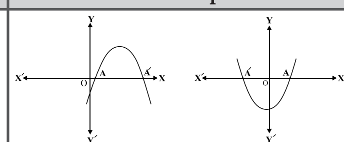
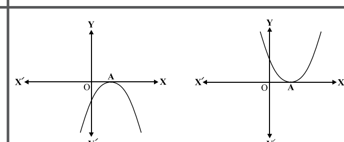
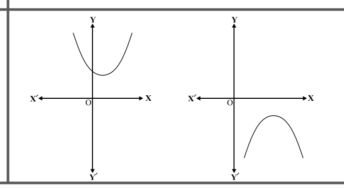
Degree of Polynomial

Highest power of x in Polynomial, p(x)

Types

Polynomial	Degree	General Form
Linear	1	ax+b
Quadratic	2	ax^2+bx+c, $a \neq 0$
Cubic	3	ax^3+bx^2+cx+d, $a \neq 0$

Zeroes of polynomial Graphically

Case	Graph	Number of Zeroes
Case1- Graph cuts x-axis at 2 points		2
Case2- Graph cuts x-axis at exactly one point		1
Case3- Graph does not cut x-axis		0

Relationship-Zeroes and Coefficient of Polynomials

Quadratic

α and β are zeroes of Quadratic Polynomial
$ax^2 + bx + c$
Then,
Sum of zeroes,
$\alpha + \beta = -\frac{b}{a}$
Product of zeroes
$\alpha\beta = \frac{c}{a}$

Cubic

α, β and ϒ are zeroes of Cubic Polynomial
$ax^3 + bx^2 + cx + d$
Sum of zeroes,
$\alpha + \beta + \Upsilon = -\frac{b}{a}$
Sum of products of the zeroes taken two at a time
$\alpha\beta + \beta\Upsilon + \Upsilon\alpha = \frac{c}{a}$
Product of zeroes
$\alpha\beta\Upsilon = -\frac{d}{a}$

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Chapter-3

Pair of Linear Equations in Two Variables

General Form

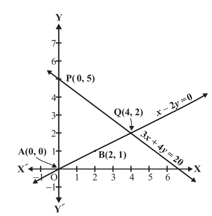
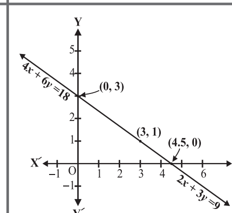
$a_1x+b_1y+c_1 = 0$
$a_2x+b_2y+c_2 = 0$

$a_1, b_1, c_1, a_2, b_2, c_2$ – Real numbers

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

Graphical Representation

S. No.	Pair of Lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the Ratios	Graphical Representation	Algebraic Interpretation
1.	$x-2y = 0$ $3x+4y-20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	 Intersecting Lines	Exactly one solution – consistent (unique)
2.	$2x+3y-9 = 0$ $4x+6y-18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	 Coincident Lines	Infinitely many solutions – Dependent

Algebraic Methods

By Substitution

Solve: $7x-15y = 2$ –(i)
$x+2y = 3$ –(ii)

Solution: From equation (ii), $x = 3-2y$-(iii)
substitute value of x in eq. (i)
$7(3-2y)-15y = 2$
$-29y = -19 \Leftrightarrow y = \frac{19}{29}$

$\therefore$ In eq. (iii) $x = 3-2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

By Elimination

Solve: $2x+3y = 8$ – (i)
$4x+6y = 7$ – (ii)

Solution: From eq. (i)x 2–eq.(ii)x1, we have
$(4x-4x) + (6y-6y) = 16-7$
$0 = 9$, which is a false statement
The pair of equation has no solution

By Cross-Multiplication

Solve: $2x+3y-46 = 0$ –(i)
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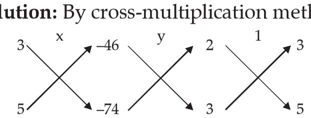
Solution: By cross-multiplication method



Then, $\frac{x}{3(-74)-5(-46)} = \frac{y}{(-46)(3)-(-74)(2)} = \frac{1}{2(5)-3(3)}$

$\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10-9}$

$\frac{x}{8} = \frac{y}{10} = \frac{1}{1} \Leftrightarrow \frac{x}{8} = \frac{1}{1}$ and $\frac{y}{10} = \frac{1}{1}$

i.e. $x = 8$ and $y = 10$

MIND MAP : LEARNING MADE SIMPLE Chapter-4

Quadratic Equations

Meaning

Equation of degree 2, in one variable

General Form

$ax^2+bx+c = 0$
a, b, c – real numbers
$a \neq 0$

Quadratic Formula

Roots of $ax^2+bx+c = 0$ are given by

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Nature of Roots

For quadratic equation $ax^2+bx+c = 0$, b^2-4ac is Discriminant (D)

S. No.	Discriminant	Roots
1.	D > 0	Two distinct real roots
2.	D = 0	Two equal real roots
3.	D < 0	No real roots (imaginary)

Solution of a Quadratic Equation

By Completing the Square

Solve: $2x^2-5x+3 = 0$

Solution: $2x^2-5x+3 = 0$

$$x^2-\frac{5}{2}x+\frac{3}{2} = 0$$

$$\left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2+\frac{3}{2} = 0 \Leftrightarrow \left(x-\frac{5}{4}\right)^2-\frac{1}{16} = 0$$

$$\left(x-\frac{5}{4}\right)^2 = \frac{1}{16} \Leftrightarrow x-\frac{5}{4} = \pm\frac{1}{4}$$

$$x = \frac{5}{4}+\frac{1}{4} \text{ or } x = \frac{5}{4}-\frac{1}{4}$$

$$x = \frac{3}{2} \text{ or } x = 1$$

By Factorization

Find roots of $6x^2-x-2=0$

Solution: $6x^2+3x-4x-2 = 0$
$3x(2x+1)-2(2x+1) = 0$
$(3x-2)(2x+1) = 0$

The roots of $6x^2-x-2=0$

$(3x-2) = 0$ or $(2x+1) = 0$

$$x = \frac{2}{3} \text{ or } x = \frac{-1}{2}$$

Roots are $\frac{2}{3}, \frac{-1}{2}$

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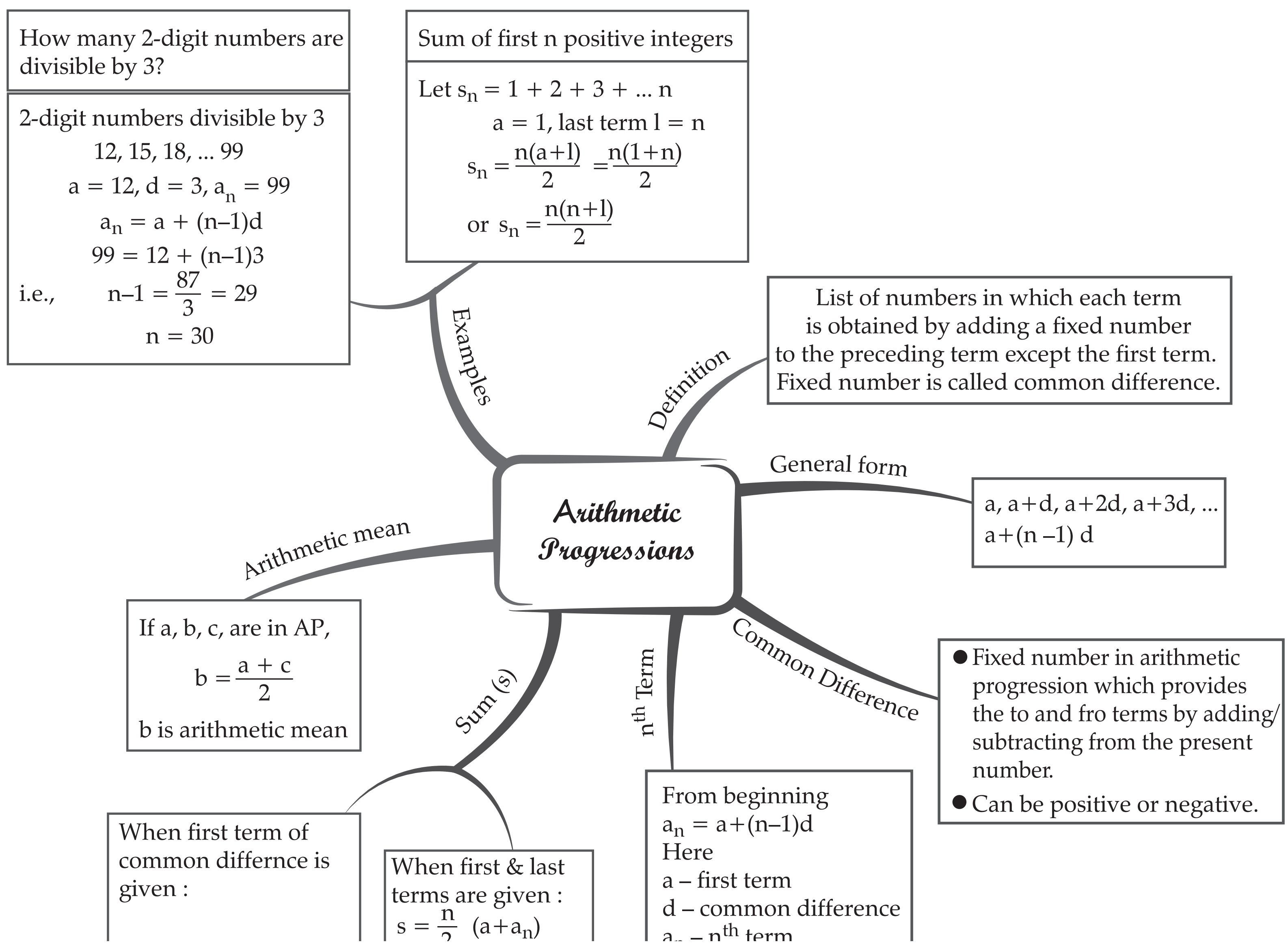
Chapter-5



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Chapter-6

Triangles

Theorems

Statement	Figure
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.	If, DE || BC then $\frac{AD}{DB} = \frac{AE}{EC}$
2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.	If $\frac{AD}{DB} = \frac{AE}{EC}$ then, DE || BC
3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)	If $\angle A = \angle D$, $\angle B = \angle E$ $\angle C = \angle F$ then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ $\Delta ABC \cong \Delta DEF$
4. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)	If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ then, $\angle A = \angle D$; $\angle B = \angle E$, $\angle C = \angle F$ $\Delta ABC \cong \Delta DEF$
5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)	If $\frac{AB}{DE} = \frac{AC}{DF}$ & $\angle A = \angle D$ then, $\Delta ABC \sim \Delta DEF$

Similarity

i) Corresponding angles are equal

ii) Corresponding sides are in the same ratio

$\Delta ABC \sim \Delta PQR$

Area of Similar Triangles

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides

Here $\Delta ABC \sim \Delta PQR$

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Right angled triangle theorem / Pythagoras

Statement	Figure
1. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse then triangles on both sides of the perpendicular are similar to the whole triangle and to each other.	In right ΔABC, $BD \perp AC$, then, $\Delta ADB \sim \Delta ABC$ $\Delta BDC \sim \Delta ABC$ $\Delta ADB \sim \Delta BDC$
2. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	In right ΔABC, $BC^2 = AB^2+AC^2$
3. In a triangle, if square of one side is equal to the sum of the squares of other two sides, then the angle opposite the first side is a right angle.	If $AC^2 = AB^2+BC^2$ then, $\angle B = 90°$

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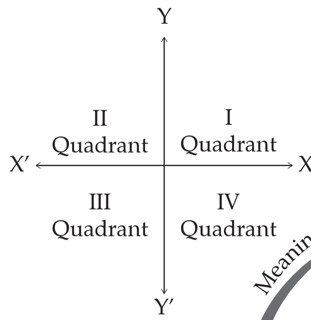
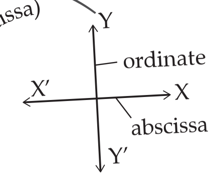
Chapter-7

Coordinate Geometry

Meaning
- Study of algebraic equations on graphs

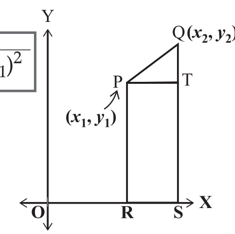


Coordinate axis
- Horizontal: x–axis (Abscissa)
- Vertical: y–axis, Ordinate



Distance formula

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Example

Are the following points vertices of a square : (1, 7), (4, 2), (–1, –1), (–4, 4)?

A (1, 7); B = (4, 2); C = (–1, –1); D = (–4, 4)

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{34}$

$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{34}$

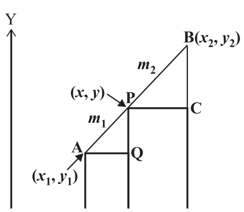
$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD.
All four sides and diagonals are equal
Hence, ABCD is a square

Section formula
- Internally (+)
- Externally (–)

$\dfrac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}, \dfrac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}$



Example

Find point of Trisection of line segment AB, A(2, –2) and B(–7, 4)

Coordinate of P (A (2, –2) — P — Q — B (–7, 4))

$= \left(\dfrac{1(-7) + 2(2)}{1+2}, \dfrac{1(4) + 2(-2)}{1+2}\right)$

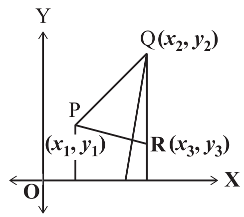
i.e., (–1, 0)

Coordinate of Q

$\left(\dfrac{2(-7) + 1(2)}{2+1}, \dfrac{2(4) + 1(-2)}{2+1}\right)$

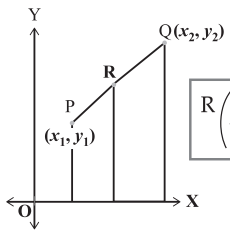
Area of Triangle

Area $= \dfrac{1}{2}\left|[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$



Mid-point Line Segment

$R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$



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Introduction to Trigonometry

Trigonometry

Study of relationships between the sides & angles of a right triangle

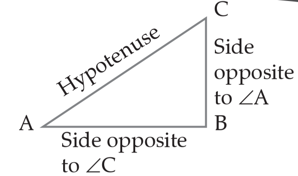
Example

Express tan A, cos A in terms of sin A
Solution : We know that, $\cos^2 A + \sin^2 A = 1$ $\cos^2 A = 1 - \sin^2 A$ ie. $\cos A = \sqrt{1 - \sin^2 A}$ $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{\sin A}{\sqrt{1 - \sin^2 A}}$

Trigonometric Identities

$\cos^2 A + \sin^2 A = 1$
$1 + \tan^2 A = \sec^2 A$ $0 \le A \le 90°$
$\cot^2 A + 1 = \text{cosec}^2 A$ $0 \le A \le 90°$

Trigonometry Ratio



Sine of ∠A	$\frac{BC}{AC}$
Cosine of ∠A	$\frac{AB}{AC}$
Tangent of ∠A	$\frac{BC}{AB}$
Cosecant of ∠A	$\frac{AC}{BC}$
Secant of ∠A	$\frac{AC}{AB}$
Cotangent of ∠A	$\frac{AB}{BC}$

Complementary Angles

$\sin (90° - A) = \cos A$
$\cos (90° - A) = \sin A$
$\tan (90° - A) = \cot A$
$\cot (90° - A) = \tan A$
$\sec (90° - A) = \text{cosec } A$
$\text{cosec } (90° - A) = \sec A$

Values

∠A	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not (∞) defined
cosec A	Not (∞) defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not (∞) defined
cot A	Not (∞) defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

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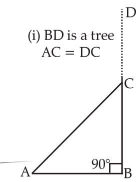
Chapter-9

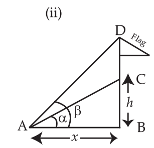
Some Application of Trigonometry

Application – Trigonometric Ratios

(To determine)

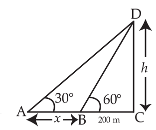
height / length of an object





Find flag length

distance between two objects

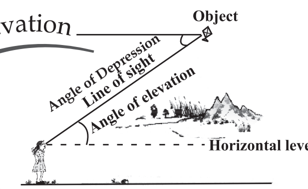


Find x and h

Measuring Angles

Angle of Elevation

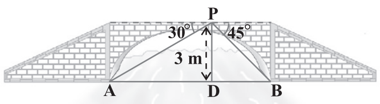
Angle of Depression



Examples

Distance

Determine width AB



From figure, AB = AD + DB

In right ΔAPD ∠A = 30°, ∠D = 90°

$$\tan 30° = \frac{PD}{AD} \text{ i.e., } AD = 3\sqrt{3}\text{m}$$

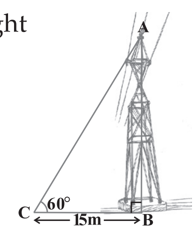
In right ΔBPD ∠B = 45°, ∠D = 90°

$$\tan 45° = \frac{PD}{BD} \text{ i.e., } BD = 3\text{m}$$

$\therefore$ AB = $(3\sqrt{3} + 3)$m = $3(\sqrt{3} + 1)$m

Object Height

Determine height of object AB



In ΔABC ∠B = 90°, ∠ C = 60°

AB

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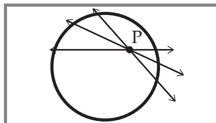
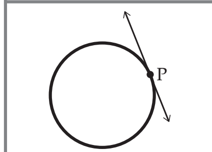
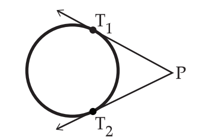
Chapter-10

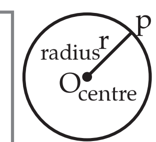
Circles

Facts

	1. There is no tangent to a circle passing through a point lying inside the circle.
	2. There is one and only one tangent to a circle passing through a point lying on the circle.
	3. There are exactly two tangents to a circle through a point lying outside the circle.

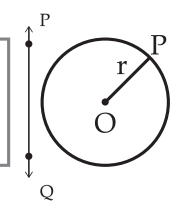
Definition

The locus of a point equidistant from a fixed point. Fixed Point is a centre & separation of points in the radius of circle.



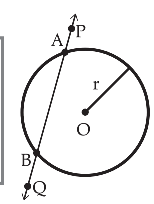
Non-intersecting line

No common point between line PQ and circle.



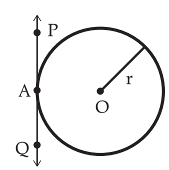
Secant

Two common points between line PQ and circle.

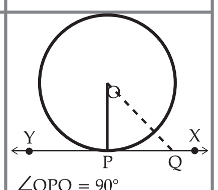
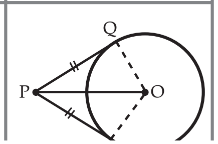


Tangent and tangent point

Only one common point between circle and PQ line.



Theorems

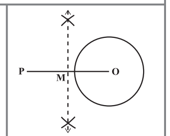
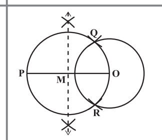
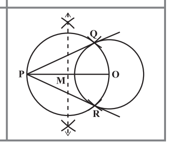
Statement	Figure
1. The tangent at any point of a circle is perpendicular to the radius through the point of contact	
2. The lengths of tangents drawn from an external point to a circle are	

MIND MAP : LEARNING MADE SIMPLE Chapter-11

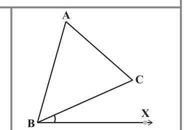
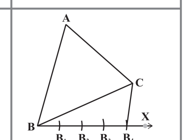
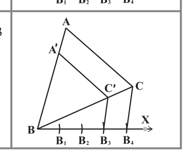
Constructions

Definition: To draw geometrical shapes using compasses, ruler etc

Tangent to circle

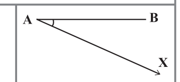
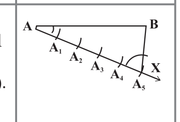
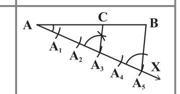
Given: Circle with centre O and point P outside it.	
1. Join PO and bisect it at mid-point M	
2. M as centre and radius = MO draw a circle, intersecting given circle at Q and R	
3. Join PQ and PR, required tangents to the circle	

Triangle similar to given triangle

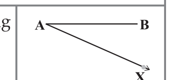
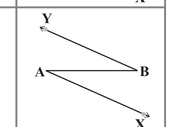
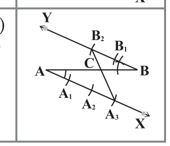
Construct a triangle similar to a given ΔABC with sides $\frac{3}{4}$ of the corresponding sides of ΔABC	
1. Draw any ray BX making an acute angle with BC	
2. Locate 4 points (greater of 3 and 4 in $\frac{3}{4}$) on BX at equal distance from each other ($BB_1 = B_1B_2 = B_2B_3 = B_3B_4$ Join B_4C	
3. Draw line parallel to B_4C from B_3 intersecting BC at C′. Draw line || to AC from C′ intersecting AB at A′ A′BC′ is the required triangle	

Line Segment — Division of line in ratio's

Method 1

Given: Line segment, ratio (3 : 2)	
1. Draw any ray AX, Making acute angle with line segment AB	
2. Locate 5 points A_1, A_2, A_3, A_4, A_5 at equal distances ($A_1 = A_2 = A_3 = A_4 = A_5$). Join BA_5	
3. Through A_3 (m = 3), draw line parallel to BA_5 cutting AB at C	 AC : CB = 3 : 2

Method 2

Given: Line segment, ratio (3 : 2)	
1. Draw any ray AX making an acute angle with line segment AB	
2. Draw ray BY || AX	
3. Locate A_1, A_2, A_3 (m = 3) on AX and B_1, B_2 (n = 2) on BY Join A_3B_2, intersecting AB at C AC : CB = 3 : 2	

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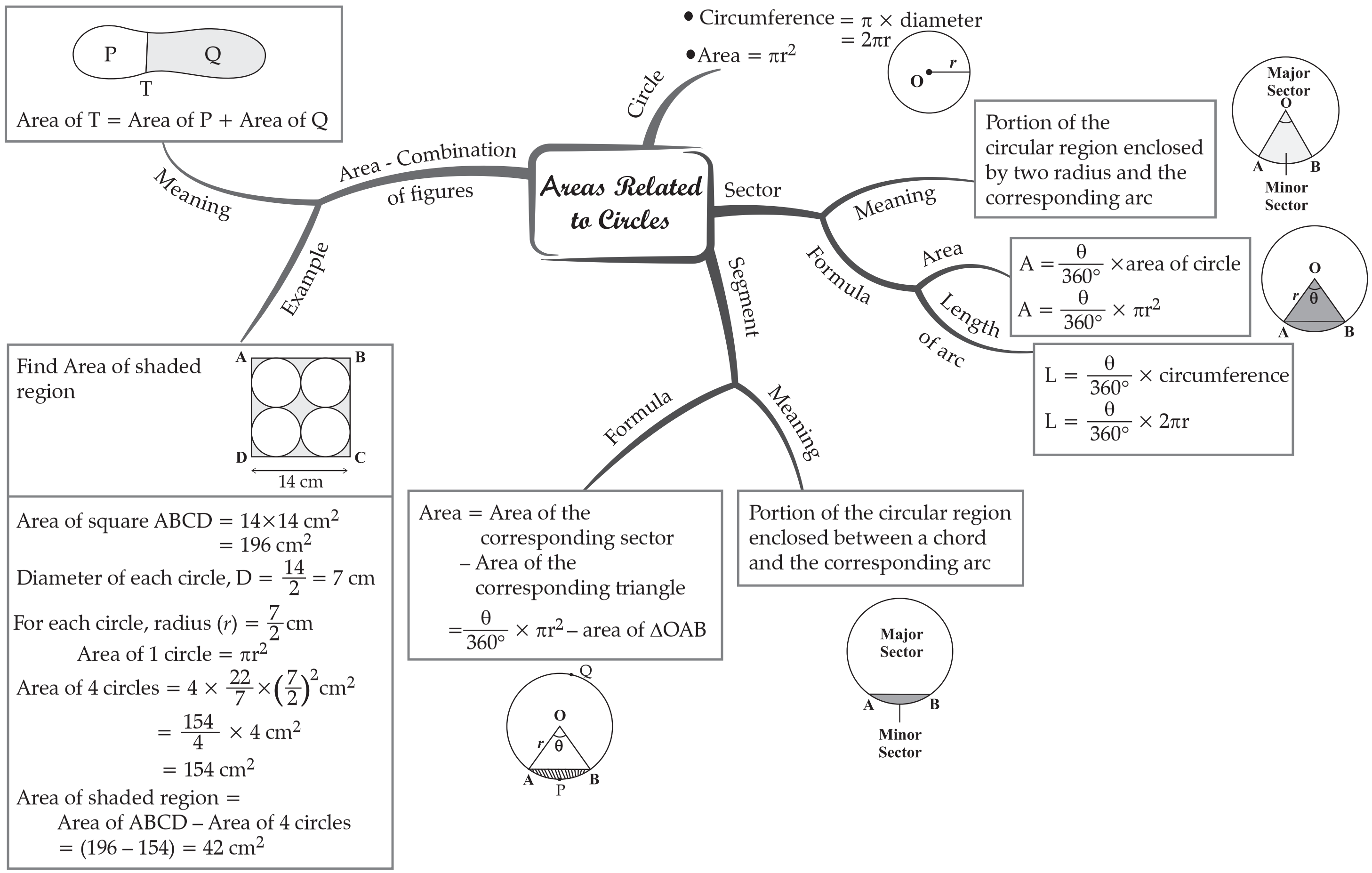
Chapter-12



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Chapter-13

Surface Areas and Volumes

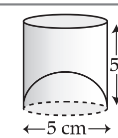
Surface Area

Sum of all of the surface areas of the faces of solid.

Volume

Quantity of 3-D space enclosed by a hollow/closed solid.

Combination of Solids



Given – Inner diameter of the Cylindrical glass = 5 cm
Height = 5 cm
Find – Actual capacity of Cylindrical glass

Solution :– Apparent capacity of the glass = $\pi r^2 h$
$= 3.14 \times 2.5 \times 2.5 \times 5 \text{ cm}^3$
$= 98.125 \text{ cm}^3$

Volume of hemisphere $= \frac{2}{3}\pi r^3$, if r=2.5cm
$= \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3$

Actual capacity = Apparent capacity – Volume of hemisphere
$= 98.125 - 32.71$
$= 65.42 \text{ cm}^3$

Conversion of Solids

A copper rod – Diameter 1cm, length 8cm converted into a wire of length 18m
Find the thickness of the wire.

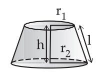
Solution : Volume of the rod $= \pi \left(\frac{1}{2}\right)^2 \times 8\text{cm}^3$
$= 2\pi \text{ cm}^3$

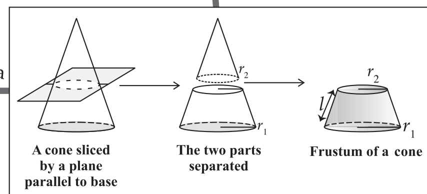
Let, r is the radius of cross-section of the wire, volume $= \pi \times r^2 \times 1800 \text{ cm}^3$

$\therefore \quad \pi \times r^2 \times 1800 = 2\pi$
$r^2 = \frac{1}{900}$
$r = \frac{1}{30}$ cm

Thickness = Diameter of the cross-section
$= \frac{1}{15}$ cm = 0.07 cm

Frustum of Cone





Curved Surface Area

$CSA = \pi l(r_1 + r_2)$
where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

Total Surface Area

$TSA = \pi l(r_1 + r_2) + \pi(r_1^2 + r_2^2)$

Volume

$V = \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

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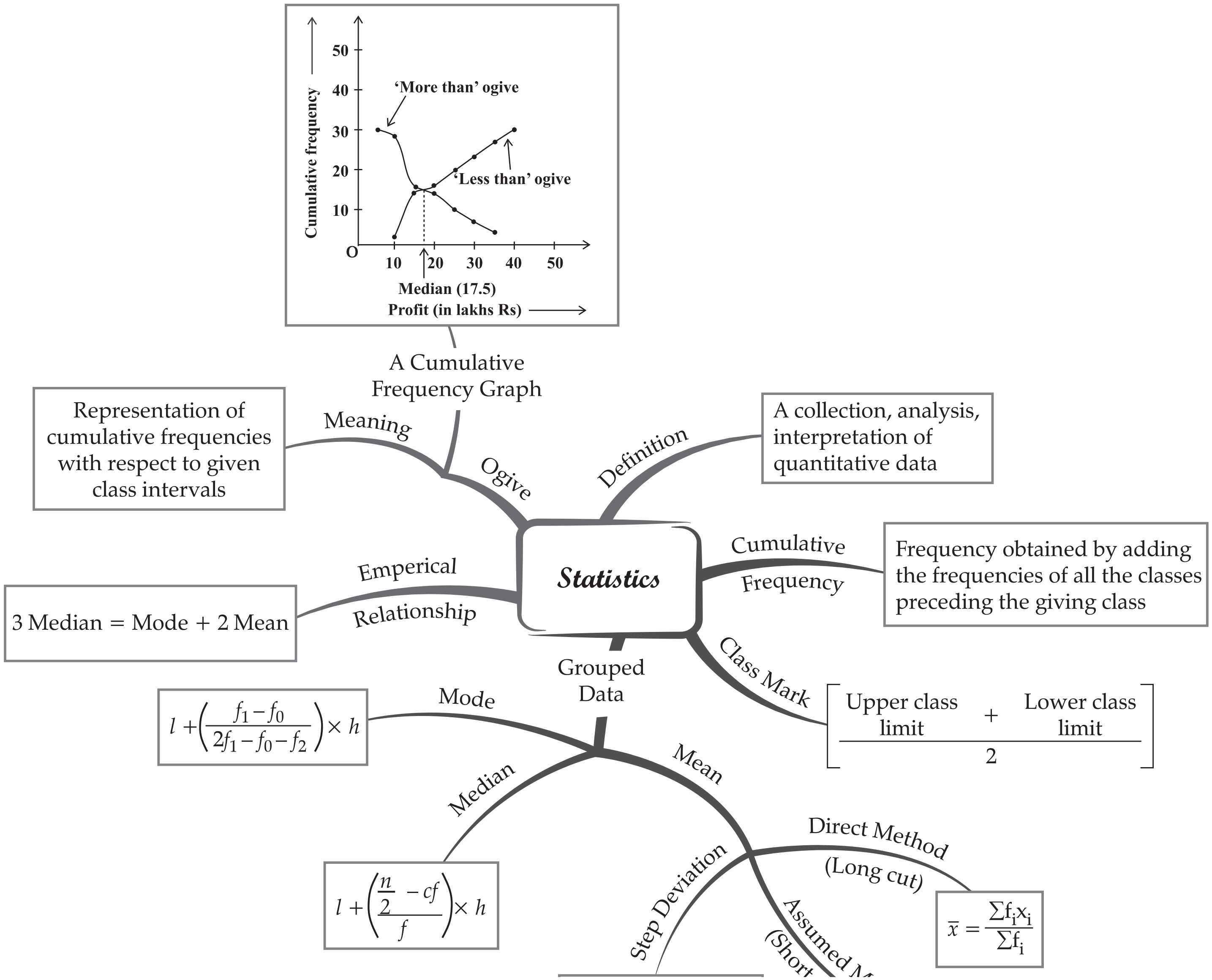
Chapter-14



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