Mind Map: Learning Made Simple

Chapter-1

Solve: 7x – 15y = 2 – (i)
x + 2y = 3 – (ii)

Solution: From equation (ii), x = 3−2y–(iii)
substitute value of x in eq. (i)
7(3−2y)−15y = 2
−29y = −19
y = \frac{19}{29}

∴ In eq. (iii) x = 3 − 2(\frac{19}{29}) = \frac{49}{29}

Solve: 2x + 3y = 8 – (i)
4x + 6y = 7 – (ii)

Solution: From eq. (i)x 2–eq.(ii)x1, we have
(4x−4x) + (6y−6y) = 16−7
0 = 9, which is a false statement
The pair of equation has no solution

Solve: 2x + 3y – 46 = 0 – (i)
3x + 5y − 74 = 0 – (ii)

Solution: By cross-multiplication method

\frac{x}{-222+230} = \frac{y}{-138+148} = \frac{1}{10−9}

i.e. x = 8 and y = 10

Solve:

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Pair of Lines</th>
<th>\frac{a_1}{a_2}</th>
<th>\frac{b_1}{b_2}</th>
<th>\frac{c_1}{c_2}</th>
<th>Compare the Ratios</th>
<th>Graphical Representation</th>
<th>Algebraic Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>x−2y = 0</td>
<td>\frac{1}{3}</td>
<td>\frac{2}{4}</td>
<td>0</td>
<td>= \frac{21−0}{42−8}</td>
<td>\text{Intersecting Lines}</td>
<td>\text{Exactly one solution}</td>
</tr>
<tr>
<td>2.</td>
<td>2x + 3y−9 = 0</td>
<td>\frac{2}{6}</td>
<td>\frac{3}{6}</td>
<td>\frac{−9}{−18}</td>
<td>= \frac{21−0}{42−8}</td>
<td>\text{Infinitely many solutions}</td>
<td>\text{Dependent}</td>
</tr>
</tbody>
</table>

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

By Substitution
By Elimination
By Cross-Multiplication
Algebraic Methods
Graphical Representation
General Form
By Substitution
By Elimination
By Cross-Multiplication
Algebraic Methods
If \( p(x) \) and \( g(x) \) are two polynomials with \( g(x) \neq 0 \), then –
\[
p(x) = g(x) \times q(x) + r(x)
\]
where, \( r(x) = 0 \) or degree of \( r(x) < \) degree of \( g(x) \)

\( \alpha \) and \( \beta \) are zeroes of Quadratic Polynomial \( ax^2 + bx + c \)
Then, Sum of zeroes,
\[
\alpha + \beta = -\frac{b}{a}
\]
Product of zeroes
\[
\alpha \beta = \frac{c}{a}
\]

\( \alpha, \beta \) and \( \gamma \) are zeroes of Cubic Polynomial \( ax^3 + bx^2 + cx + d \)
Sum of zeroes,
\[
\alpha + \beta + \gamma = -\frac{b}{a}
\]
Sum of products of the zeroes taken two at a time
\[
\alpha \beta + \beta \gamma + \gamma \alpha = \frac{c}{a}
\]
Product of zeroes
\[
\alpha \beta \gamma = -\frac{d}{a}
\]

<table>
<thead>
<tr>
<th>Case</th>
<th>Graph</th>
<th>Number of Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1- Graph cuts ( x )-axis at 2 points</td>
<td>![Graph1]</td>
<td>2</td>
</tr>
<tr>
<td>Case2- Graph cuts ( x )-axis at exactly one point</td>
<td>![Graph2]</td>
<td>1</td>
</tr>
<tr>
<td>Case3- Graph does not cut ( x )-axis</td>
<td>![Graph3]</td>
<td>0</td>
</tr>
</tbody>
</table>

**Types of Polynomial**

<table>
<thead>
<tr>
<th>Polynomial</th>
<th>Degree</th>
<th>General Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>1</td>
<td>( ax + b )</td>
</tr>
<tr>
<td>Quadratic</td>
<td>2</td>
<td>( ax^2 + bx + c ) ( a \neq 0 )</td>
</tr>
<tr>
<td>Cubic</td>
<td>3</td>
<td>( ax^3 + bx^2 + cx + d ) ( a \neq 0 )</td>
</tr>
</tbody>
</table>
Chapter 3: Pair of Linear Equations in Two Variables

General Form

- By Elimination
- By Cross-Multiplication
- By Substitution

Solve: \[ \begin{align*}
7x - 15y &= 2 \quad \text{--(i)} \\
x + 2y &= 3 \quad \text{--(ii)} 
\end{align*} \]

**Solution:** From equation (ii), \( x = 3 - 2y \) (iii)
Substitute value of \( x \) in eq. (i)

\[ \begin{align*}
7(3-2y) - 15y &= 2 \\
21 - 14y - 15y &= 2 \\
-29y &= -19 \\
y &= \frac{19}{29}
\end{align*} \]

\[ x = 3 - 2 \left( \frac{19}{29} \right) = \frac{49}{29} \]

Solve: \( 2x + 3y = 8 \) \( \text{--(i)} \)
\( 4x + 6y = 7 \) \( \text{--(ii)} \)

**Solution:** From eq. (i)x 2 – eq. (ii)x 1, we have

\( 4x - 4x + 6y - 6y = 16 - 7 \)

\( 0 = 9 \), which is a false statement

The pair of equation has no solution

Solve: \( 2x + 3y - 46 = 0 \) \( \text{--(i)} \)
\( 3x + 5y - 74 = 0 \) \( \text{--(ii)} \)

**Solution:** By cross-multiplication method

\[ \begin{align*}
x &= \frac{46 \times 3 - 5 \times 46}{3 \times 5 - 74} \\
5 &= \frac{-222 + 230}{-138 + 148} = \frac{1}{10} - 9 \\
\end{align*} \]

\[ x = \frac{8}{10} = \frac{4}{5} \]

Then, \( y = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5} \)

i.e. \( x = 8 \) and \( y = 10 \)

Solve: \( 7x - 15y = 2 \) \( \text{--(i)} \)
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**Solve:**

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y &= \frac{19}{29}
\end{align*} \]

\[ x = 3 - 2 \left( \frac{19}{29} \right) = \frac{49}{29} \]

- If \( a_1 \neq a_2 \) and \( b_1 \neq b_2 \) and \( c_1 \neq c_2 \), then the pair of equations has no solution.
- If \( a_1 = a_2 \) and \( b_1 = b_2 \) and \( c_1 \neq c_2 \), then the pair of equations is inconsistent.
- If \( a_1 = a_2 \) and \( b_1 = b_2 \) and \( c_1 = c_2 \), then the pair of equations is consistent and has infinitely many solutions.
- If \( a_1 \neq a_2 \) and \( b_1 = b_2 \) and \( c_1 = c_2 \), then the pair of equations is consistent and has unique solution.

**Graphical Interpretation:**

- **Intersecting Lines**
  - Exactly one solution – consistent (unique)
- **Coincident Lines**
  - Infinitely many solutions – Dependent
- **Parallel Lines**
  - No solution – Inconsistent
Chapter 4: Quadratic Equations

Solution of a Quadratic Equation

By Completing the Square

Meaning

Equation of degree 2, in one variable

General Form

\[ ax^2 + bx + c = 0 \]

\( a, b, c \) - real numbers

For quadratic equation \( ax^2 + bx + c = 0 \),

\( b^2 - 4ac \) is Discriminant (D)

Nature of Roots

The roots of \( ax^2 + bx + c = 0 \) are given by

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Solve: \( 2x^2 - 5x + 3 = 0 \)

Solution:

\[ 2x^2 - 5x + 3 = 0 \]

\[ 2x^2 - \frac{5}{2} x + \frac{3}{2} = 0 \]

\( \left( x - \frac{5}{4} \right)^2 = \left( \frac{5}{4} \right)^2 - \frac{3}{2} = 0 \iff \left( x - \frac{5}{4} \right)^2 = \frac{1}{16} = 0 \]

\[ x = \frac{5}{4} \pm \frac{1}{4} \]

\[ x = \frac{5}{4} + \frac{1}{4} \text{ or } x = \frac{5}{4} - \frac{1}{4} \]

\[ x = \frac{6}{4} \text{ or } x = 1 \]

Find roots of \( 6x^2 - x - 2 = 0 \)

Solution:

\[ 6x^2 + 3x - 4x - 2 = 0 \]

\[ 3x(2x + 1) - 2(2x + 1) = 0 \]

\[ (3x - 2)(2x + 1) = 0 \]

The roots of \( 6x^2 - x - 2 = 0 \)

\[ (3x - 2) = 0 \text{ or } (2x + 1) = 0 \]

\[ x = \frac{2}{3} \text{ or } x = -\frac{1}{2} \]

Roots are \( \frac{2}{3} \) and \( -\frac{1}{2} \)

Table of Discriminant and Roots

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Discriminant</th>
<th>Roots</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D &gt; 0</td>
<td>Two distinct real roots</td>
</tr>
<tr>
<td>2</td>
<td>D = 0</td>
<td>Two equal real roots</td>
</tr>
<tr>
<td>3</td>
<td>D &lt; 0</td>
<td>No real roots (imaginary)</td>
</tr>
</tbody>
</table>
Arithmetic Progressions

**Definition**
List of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. Fixed number is called common difference.

**General form**
- \( a, a+d, a+2d, a+3d, \ldots\)
- \( a+(n-1)d\)

**From beginning**
- \( a_n = a+(n-1)d\)
- Here
  - \( a \) – first term
  - \( d \) – common difference

**When first term of common difference is given:**
- \( s = \frac{n}{2} (a+a_n)\)

**When first & last terms are given:**
- \( s = \frac{n}{2} \) \((a+a_n)\)

**2-digit numbers divisible by 3**
- 12, 15, 18, ..., 99
- \( a = 12, d = 3, a_n = 99\)
- \( 99 = 12 + (n-1)3 \)
- \( n-1 = \frac{87}{3} = 29\)
- \( n = 30\)

**Sum of first \( n \) positive integers**
Let \( s_n = 1 + 2 + 3 + \ldots n\)
- \( a = 1, \) last term \( l = n\)
- \( s_n = \frac{n(a+l)}{2} = \frac{n(1+n)}{2}\)
- or \( s_n = \frac{n(n+1)}{2}\)

**Examples**
- If \( a, b, c, \) are in AP,
  - \( b = \frac{a+c}{2}\)
  - \( b \) is arithmetic mean

- How many 2-digit numbers are divisible by 3?

- Sum of first \( n \) positive integers

- List of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. Fixed number is called common difference.

- Fixed number in arithmetic progression which provides the to and fro terms by adding/subtracting from the present number.

- Can be positive or negative.
Chapter-6

Triangles

**Theorem**

1. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then triangles on both sides of the perpendicular are similar to the whole triangle and to each other. 

\[ \triangle ADB \sim \triangle ABC \]
\[ \triangle BDC \sim \triangle ABC \]

2. In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

\[ AC^2 = AB^2 + BC^2 \]

3. In a triangle, if square of one side is equal to the sum of the squares of other two sides, then the angle opposite the first side is a right angle.

**Similarity**

1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

\[ \frac{AD}{DB} = \frac{AE}{EC} \]

2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar. (AAA criterion)

4. If in two triangles, sides of one triangle are proportional to (i.e., in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar. (SSS criterion)

5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar. (SAS criterion)

**Pythagoras**

\[ AB^2 + BC^2 = AC^2 \]

**Area of Similar Triangles**

\[ \frac{ar(\triangle ABC)}{ar(\triangle PQR)} = \left( \frac{AB}{QR} \right)^2 \]
**Chapter-7**

**Coordinate Geometry**

**Mind Map: Learning Made Simple**

- **Meaning**: Study of algebraic equations on graphs.

- **Coordinate axis**
  - **Horizontal**: (Abscissa)
  - **Vertical**: (Ordinate)

- **Distance formula**

- **Section formula**

- **Mid-point Line Segment**

- **Area of Triangle**

- **Example**

- **Example**

- **Find point of Trisection of line segment AB, A(2, -2) and B(-7, 4)**

- **Coordinate of P**

- **Coordinate of Q**

- **Area**

- **Are the following points vertices of a square: (1, 7), (4, 2), (-1, -1), (-4, 4)?**

- **A (1, 7): B = (4, 2); C = (-1, -1); D = (-4, 4)**

- **AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{34}**

- **BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{34}**

- **CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{34}**

- **DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{34}**

- **AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{68}**

- **BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{68}**

- **Since, AB = BC = CD = DA and AC = BD. All four sides and diagonals are equal. Hence, ABCD is a square.**
Study of relationships between the sides & angles of a right triangle.

**Introduction to Trigonometry**

**Trigonometric Identities**

- \( \cos^2 A + \sin^2 A = 1 \)
- \( 1 + \tan^2 A = \sec^2 A \) \( 0 \leq A \leq 90^\circ \)
- \( \cot^2 A + 1 = \cosec^2 A \) \( 0 \leq A \leq 90^\circ \)

**Example**

Express \( \tan A, \cos A \) in terms of \( \sin A \)

**Solution**:

We know that, \( \cos^2 A + \sin^2 A = 1 \)

\[
\cos^2 A = 1 - \sin^2 A \rightarrow \cos A = \sqrt{1 - \sin^2 A}
\]

**Trigonometry Ratio**

- Sine of \( \angle A \)
- Cosine of \( \angle A \)
- Tangent of \( \angle A \)
- Cosecant of \( \angle A \)
- Secant of \( \angle A \)
- Cotangent of \( \angle A \)

**Values**

- \( \sin 0^\circ = 0 \)
- \( \sin 30^\circ = \frac{1}{2} \)
- \( \sin 45^\circ = \frac{\sqrt{2}}{2} \)
- \( \sin 60^\circ = \frac{\sqrt{3}}{2} \)
- \( \sin 90^\circ = 1 \)

- \( \cos 0^\circ = 1 \)
- \( \cos 30^\circ = \frac{\sqrt{3}}{2} \)
- \( \cos 45^\circ = \frac{\sqrt{2}}{2} \)
- \( \cos 60^\circ = \frac{1}{2} \)
- \( \cos 90^\circ = 0 \)

- \( \tan 0^\circ = 0 \)
- \( \tan 30^\circ = \frac{\sqrt{3}}{3} \)
- \( \tan 45^\circ = 1 \)
- \( \tan 60^\circ = \sqrt{3} \)
- \( \tan 90^\circ \) not defined

- \( \cot 0^\circ \) not defined
- \( \cot 30^\circ = \sqrt{3} \)
- \( \cot 45^\circ = 1 \)
- \( \cot 60^\circ = \frac{1}{\sqrt{3}} \)
- \( \cot 90^\circ \) not defined

**Complementary Angles**

- \( \sin (90^\circ - A) = \cos A \)
- \( \cos (90^\circ - A) = \sin A \)
- \( \tan (90^\circ - A) = \cot A \)
- \( \cot (90^\circ - A) = \tan A \)
- \( \sec (90^\circ - A) = \cosec A \)
- \( \cosec (90^\circ - A) = \sec A \)

**Important Formulas**

- \( 1 + \tan^2 A = \sec^2 A \) \( 0 \leq A \leq 90^\circ \)
- \( \cot^2 A + 1 = \cosec^2 A \) \( 0 \leq A \leq 90^\circ \)

**Trigonometry Values**

<table>
<thead>
<tr>
<th>( \angle A )</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin A )</td>
<td>0</td>
<td>1/2</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>( \cos A )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1/2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( \tan A )</td>
<td>0</td>
<td>1/( \sqrt{3} )</td>
<td>1</td>
<td>( \sqrt{3} )</td>
<td>Not (( \infty )) defined</td>
</tr>
<tr>
<td>( \csc A )</td>
<td>Not (( \infty )) defined</td>
<td>2</td>
<td>( \sqrt{2} )</td>
<td>( \frac{2}{\sqrt{3}} )</td>
<td>1</td>
</tr>
<tr>
<td>( \sec A )</td>
<td>1</td>
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<tr>
<td>( \cot A )</td>
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<td>( \sqrt{3} )</td>
<td>1</td>
<td>( \frac{1}{\sqrt{3}} )</td>
<td>0</td>
</tr>
</tbody>
</table>
Chapter-9

Some Application of Trigonometry

Determine width $AB$

From figure, $AB = AD + DB$
In right $\triangle APD$, $\angle A = 30^\circ$, $\angle D = 90^\circ$
$\tan 30^\circ = \frac{PD}{AD}$ i.e., $AD = 3\sqrt{3}m$
In right $\triangle BPD$, $\angle B = 45^\circ$, $\angle D = 90^\circ$
$\tan 45^\circ = \frac{PD}{BD}$ i.e., $BD = 3m$
$\therefore AB = (3\sqrt{3} + 3)m = 3(\sqrt{3} + 1)m$

Determine height of object $AB$

In $\triangle ABC$, $\angle B = 90^\circ$, $\angle C = 60^\circ$

Angle of Depression

Angle of Elevation

Distance

Examples

Object Height

Measuring Angles

Angle of Elevation

Angle of Depression

Height / length of an object

Distance between two objects

Find flag length $x$ and $h$

Find $x$ and $h$
**Circles**

**Definition**

The locus of a point equidistant from a fixed point. Fixed Point is a centre & separation of points in the radius of circle.

**Facts**

1. There is no tangent to a circle passing through a point lying inside the circle.
2. There is one and only one tangent to a circle passing through a point lying on the circle.
3. There are exactly two tangents to a circle through a point lying outside the circle.

**Theorems**

1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. The lengths of tangents drawn from an external point to a circle are equal.
3. There is no tangent to a circle passing through a point lying inside the circle.
4. There is one and only one tangent to a circle passing through a point lying on the circle.
5. There are exactly two tangents to a circle through a point lying outside the circle.

**Statement**

<table>
<thead>
<tr>
<th>Statement</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.</td>
<td>![Diagram of tangent and radius]</td>
</tr>
<tr>
<td>2. The lengths of tangents drawn from an external point to a circle are equal.</td>
<td>![Diagram of tangents from external point]</td>
</tr>
</tbody>
</table>

**Non-intersecting line**

No common point between line PQ and circle.

**Tangent and tangent point**

Only one common point between circle and PQ line.

**Secant**

Two common points between line PQ and circle.

---

Oswaal CBSE Chapterwise Mind Maps, MATHEMATICS, Class-X
Chapter 11: Constructions

**Definition**
To draw geometrical shapes using compasses, ruler etc.

**Line Segment Division**
1. Draw any ray AX, making acute angle with line segment AB
2. Locate 5 points A, A1, A2, A3, A4, A5 at equal distances (A1 = A2 = A3 = A4 = A5).
3. Join BA5
4. Through A3 (m = 3), draw line parallel to BA cutting AB at C

**Tangent to circle**
1. Draw any ray AX, making an acute angle with BC
2. Locate 4 points (greater of 3 and 4 in ) on BX at equal distance from each other (B1 = B1 B2 = B2 B3 = B3 B4)
3. Join B4 C
4. Draw line parallel to B4 C from B3 intersecting BC at C'
5. Draw line parallel to AC from C' intersecting AB at A'

**Construct a triangle similar to a given ΔABC with sides of the corresponding sides of ΔABC**
1. Draw any ray BX making an acute angle with BC
2. Locate 4 points (greater of 3 and 4 in ) on BX at equal distance from each other (B1 = B1 B2 = B2 B3 = B3 B4)
3. Join B4 C
4. Draw line parallel to B4 C from B3 intersecting BC at C'
5. Draw line parallel to AC from C' intersecting AB at A'

**Given: Circle with centre O and point P outside it.**
1. Join PO and bisect it at mid-point M
2. M as centre and radius = MO, draw a circle, intersecting given circle at Q and R
3. Join PQ and PR, required tangents to the circle
Areas Related to Circles

- **Meaning**
  - **Area**
  - **Length of arc**
  - **Segment** Formula

**Portion of the circular region enclosed by two radius and the corresponding arc**

\[
A = \frac{\theta}{360} \times \text{Area of circle}
\]

\[
A = \frac{\theta}{360} \times \pi r^2
\]

**Length of arc**

\[
L = \frac{\theta}{360} \times 2\pi r
\]

**Example**

- **Portion of the circular region enclosed between a chord and the corresponding arc**

\[
\text{Area} = \text{Area of the corresponding sector} - \text{Area of the corresponding triangle}
\]

\[
= \frac{\theta}{360} \times \pi r^2 - \text{area of } \triangle OAB
\]

**Find Area of shaded region**

- **Area of square ABCD**

\[
14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2
\]

- **Diameter of each circle**

\[
D = \frac{14}{2} = 7 \text{ cm}
\]

- **For each circle, radius**

\[
r = \frac{7}{2} \text{ cm}
\]

- **Area of 1 circle**

\[
\pi \left(\frac{7}{2}\right)^2 = \frac{49\pi}{4} \text{ cm}^2
\]

- **Area of 4 circles**

\[
4 \times \frac{49\pi}{4} = 49\pi \text{ cm}^2
\]

- **Area of shaded region**

\[
196 - 49\pi 
\]

- **Area of shaded region**

\[
= \frac{154}{\pi} \text{ cm}^2
\]

**Area of ABCD**

\[
196 - 154 = 42 \text{ cm}^2
\]
Chapter-13

**Surface Areas and Volumes**

- **Total Surface Area (TSA):**
  
  \[ TSA = \pi l(r_1 + r_2) + \pi (r_1^2 + r_2^2) \]

- **Curved Surface Area (CSA):**
  \[ CSA = \pi l(r_1 + r_2) \]

  where \( l = \sqrt{h^2 + (r_1 - r_2)^2} \)

- **Volume (V):**
  \[ V = \frac{1}{3} \pi r^2 h \]

A copper rod – Diameter 1cm, length 8cm converted into a wire of length 18m

Find the thickness of the wire.

**Solution:**

Volume of the rod = \( \pi \left( \frac{1}{2} \right)^2 \times 8 \text{cm}^3 = 2\pi \text{cm}^3 \)

Let, \( r \) is the radius of cross-section of the wire, volume = \( \pi r^2 \times 1800 \text{ cm}^3 \)

\[ \pi r^2 \times 1800 = 2\pi \]

\[ r^2 = \frac{1}{900} \]

\[ r = \frac{1}{30} \text{ cm} \]

Thickness = Diameter of the cross-section

\[ = \frac{1}{15} \text{ cm} = 0.07 \text{ cm} \]

**Sum of all of the surface areas of the faces of solid.**

**Quantity of 3-D space enclosed by a hollow/closed solid.**

**Combination of Solids**

**Conversion of Solids**

**A cone sliced by a plane parallel to base**

**The two parts separated**

**Frustum of a cone**

**A hemisphere – Radius 2.5cm**

Volume of hemisphere = \( \frac{2}{3} \pi r^3 \text{ if } r=2.5 \text{ cm} = \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3 \)

**Given – Inner diameter of the Cylindrical glass = 5 cm**

**Find – Actual capacity of Cylindrical glass**

**Solution:**

Apparent capacity of the glass = \( \pi h \)

= \( 3.14 \times 2.5 \times 2.5 \times 5 \text{ cm}^3 = 98.125 \text{ cm}^3 \)

Volume of hemisphere = \( \frac{2}{3} \pi r^3 \)

= \( \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3 \)

Actual capacity = Apparent capacity – Volume of hemisphere

= 98.125 – 32.71

= 65.42 \text{ cm}^3
Chapter-14
Statistics

Definition
Cumulative Frequency
Mean (Class Mark) (Short cut)
Direct Method (Long cut)
Assumed Mean
Step Deviation
Median
Ogive

A collection, analysis, interpretation of quantitative data
Frequency obtained by adding the frequencies of all the classes preceding the giving class
Upper class limit + Lower class limit
Class Mark
Mean
Mode

\[
\text{Grouped Data} = \frac{\sum fx}{\sum f} = a + \frac{\sum fi f_i}{\sum f}
\]

\[
x = a + \left(\frac{\sum f_i u_i}{\sum f} \times h\right)
\]

\[
\text{Median} = \text{Mode} + 2 \times \text{Mean}
\]

A Cumulative Frequency Graph
Representation of cumulative frequencies with respect to given class intervals

3 Median = Mode + 2 Mean

\[
\text{Mean} = \frac{\sum f_i x_i}{\sum f_i}
\]

\[
\text{Frequency} = \sum f_i
\]
Chapter 15: Probability

**Theoretical Probability**
- What we expect to happen in an experiment
- Number of trials in which the event happened
- Number of favorable outcomes
- Total number of trials

**Experimental Probability**
- What actually happens in an experiment
- Number of outcomes favorable to E
- Number of all possible outcomes of the experiment

**Definitions**
- Elementary Event
- Sure or Certain Event
- Complementary Event

**Examples**
- Coin:
  - When a coin is tossed, what would be the probability of appearing head?
  - Solution: Total outcomes = 2
  - Favorable outcomes = 1
  - Required Prob. \( P(E) = \frac{1}{2} \)

- Card:
  - What is the probability of getting an ace from a pack of 52 cards?
  - Solution: Number of favorable outcomes = 4
  - Number of possible outcomes = 52
  - \( P(E) = \frac{4}{52} = \frac{1}{13} \)

- Dice:
  - Two dice are rolled, what is probability of getting 13 as a sum?
  - Solution: Number of possible outcomes = \( 6^2 = 36 \)
  - Number of favorable outcomes = 0
  - As addition of no two number on the dice will give sum = 13 is not possible
  - \( P(E) = \frac{0}{36} = 0 \)

- Dice and Coin:
  - When a coin is tossed and dice is rolled, what is the probability of getting head on coin and at least one 6 on dice?
  - Solution: Total favorable outcomes = 6
  - Total possible outcomes = 72
  - \( P(E) = \frac{6}{72} = \frac{1}{12} \)

**Important Results**
- Sum of probabilities of all elementary events is 1.
  - For events A, B, C: \( P(A) + P(B) + P(C) = 1 \)

**Complementary Event**
- Event having probability to occur as 1
  - For event E, complement event, \( P(\overline{E}) = 1 - P(E) \)